Lecture 7: Supervised Learning Pt. 1

Linear Classifiers and Cross Validation
INFO 1998: Introduction to Machine Learning

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Announcements

- **Assignment 6:** Due tonight at 11:59pm
- **Assignment 7:** Due next Monday at 11:59pm
- **Mid-Semester Check-Ins:** Due today! Please get them done.
- **Next Lecture:** More Supervised Learning! (Decision Trees & Logistic Regression)

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Agenda

1. Linear Classifiers
    - Linear Perceptron
    - Support Vector Machines (SVMs)
2. Cross Validation (K-Fold)

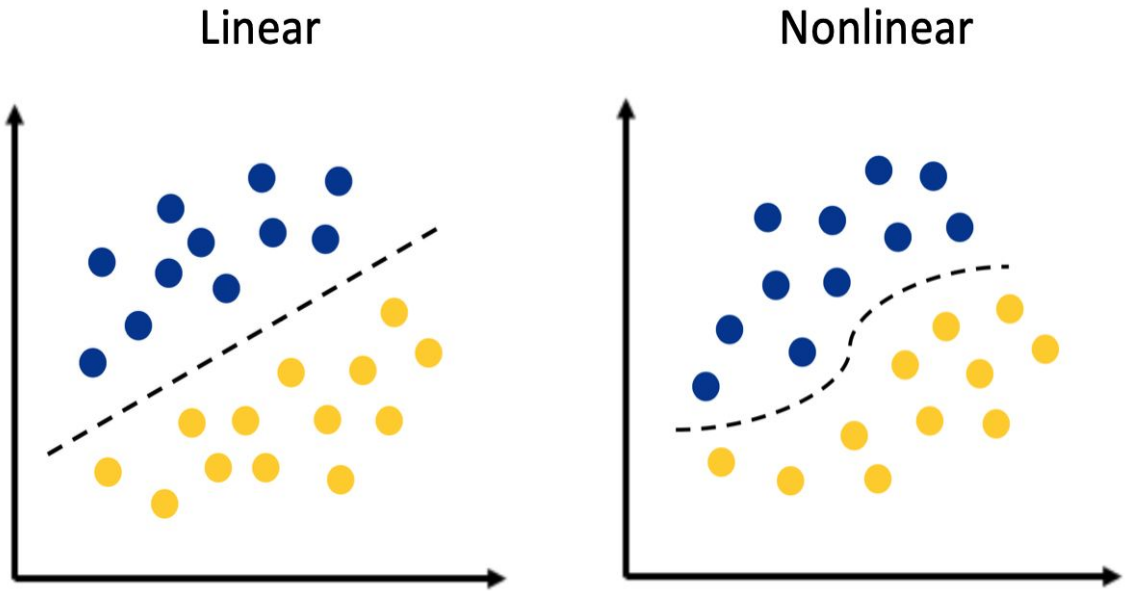
Linear Classifiers

Source

Linear Classifiers

A linear classifier is a hyper plane that is used to classify our data points

A hyperplane is our **decision boundary** and our goal is to find the best hyper plane for our data.

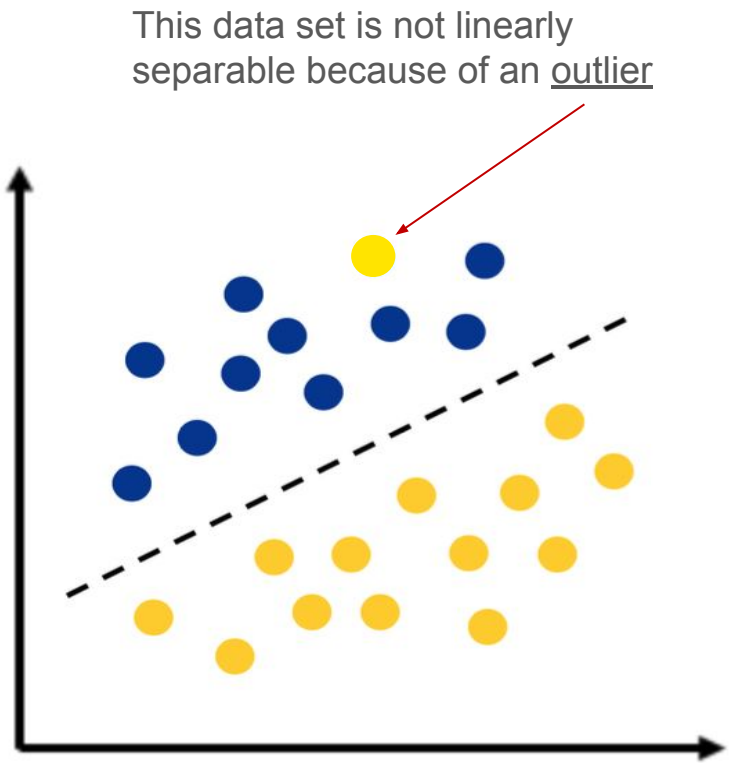


Source

Linearly Separable

In this example, we cannot partition our dataset into yellow and purple with a linear decision boundary. This means that our data is not **linearly separable.**

Outliers are frequently the reason a data set is not linearly separable.



Perceptron Learning Algorithm

Goal: find a normal vector w that perfectly classifies all the points in our data set
Algorithm:

```
Initialize classifier as some random hyperplane
While there exists a misclassified point x:
        Adjust classifier slightly so that it classifies x correctly
                (or, is a little closer to classifying x correctly)
End While
```

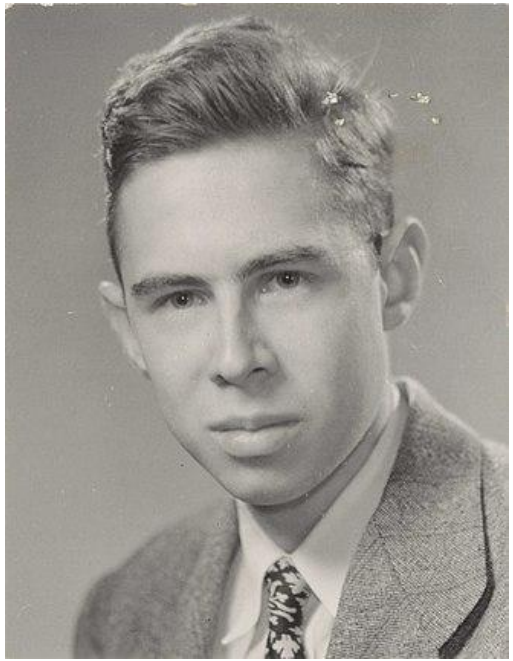
“Use your mistakes as your stepping stones”

History of the Linear Perceptron

Frank Rosenblatt was first to implement perceptron!
→ Cornell professor and alum PHD ‘56 🐻❗

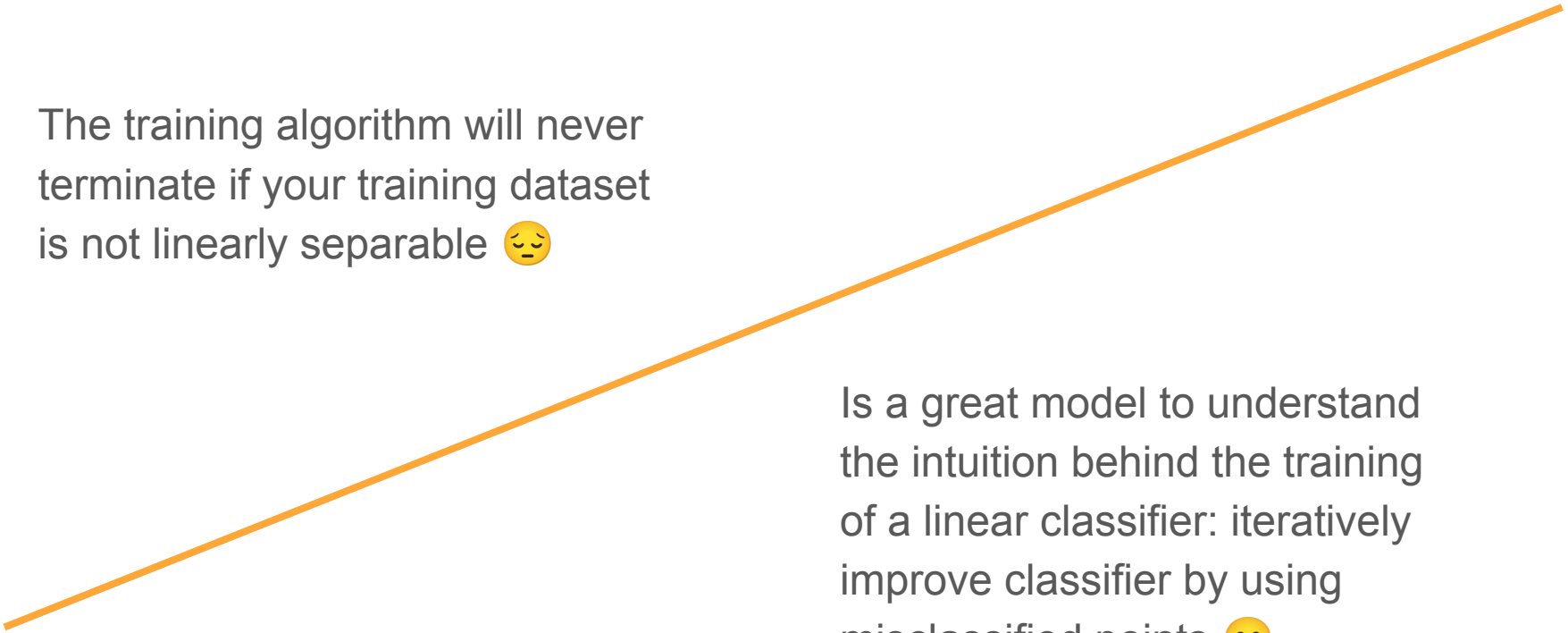
Gave him the title of ‘Father of Deep Learning’

Deep Learning
→ Neural Networks a.k.a. Multilayer Perceptrons



Limitations of Perceptron

The training algorithm will never terminate if your training dataset is not linearly separable 😔



Is a great model to understand the intuition behind the training of a linear classifier: iteratively improve classifier by using misclassified points 😄

Perceptron

Algorithm

Initialize $\vec{w} = \vec{0}$ // Initialize $\vec{w}$. $\vec{w} = \vec{0}$ misclassifies everything.
while TRUE **do** // Keep looping
    $m = 0$ // Count the number of misclassifications, m
    for $(x_i, y_i) \in D$ **do** // Loop over each (data, label) pair in the dataset,
        if $y_i(\vec{w}^T \cdot \vec{x_i}) \leq 0$ **then** // If the pair $(\vec{x_i}, y_i)$ is misclassified
            $\vec{w} \leftarrow \vec{w} + y\vec{x}$ // Update the weight vector $\vec{w}$
            $m \leftarrow m + 1$ // Counter the number of misclassification
        end if
    end for
    if $m = 0$ **then** // If the most recent $\vec{w}$ gave 0 misclassifications
        break // Break out of the while-loop
    end if
end while // Otherwise, keep looping!

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Linear Classifiers and Cross Validation
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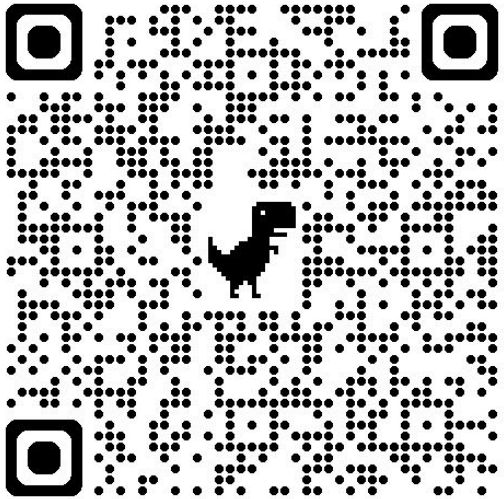

Attendance!

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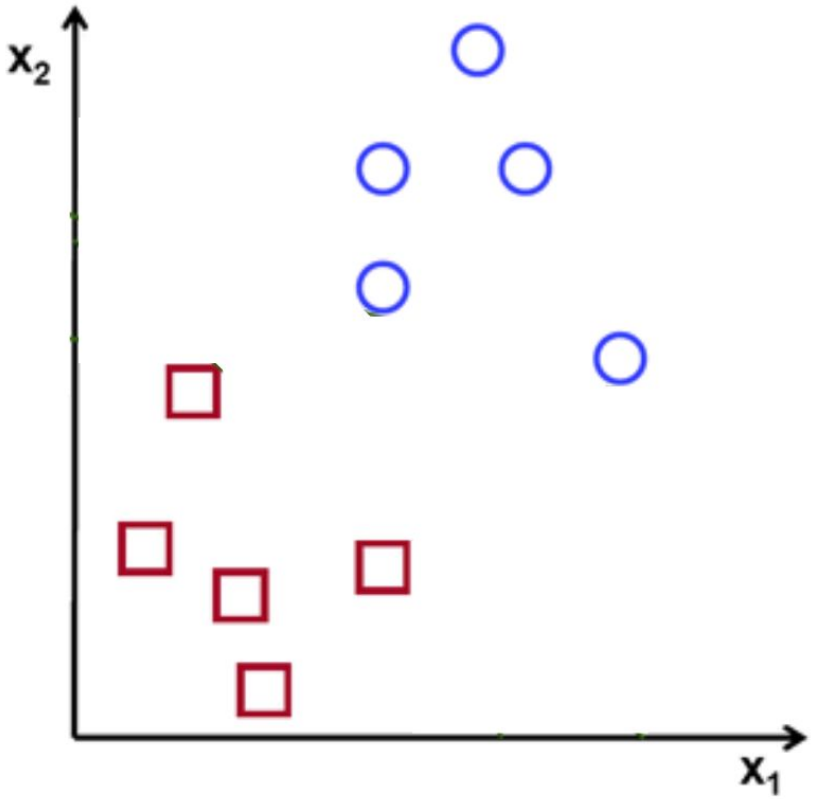
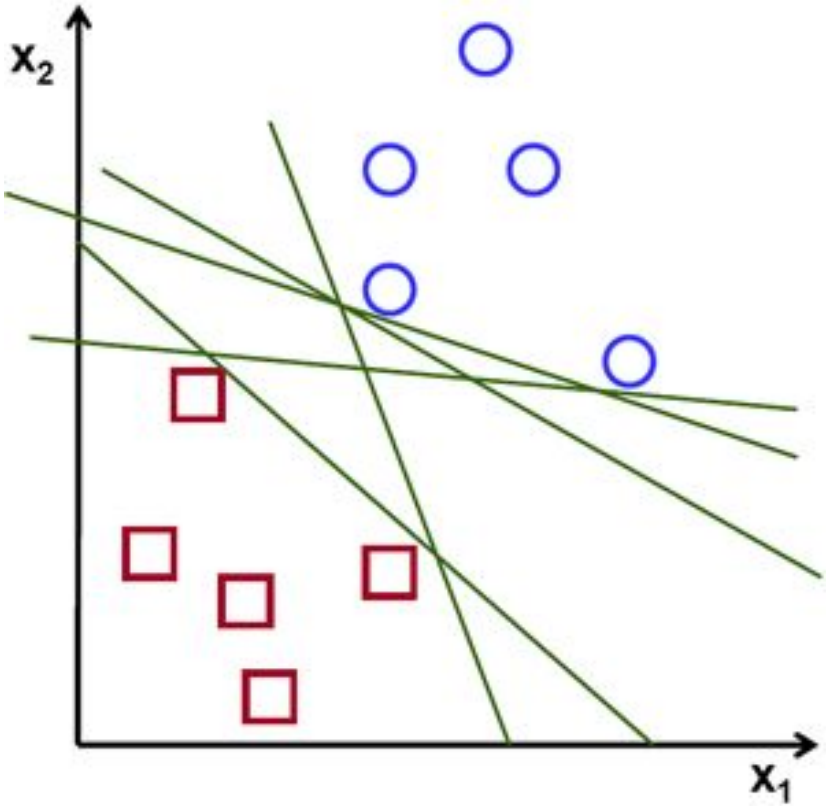
Support Vector Machines

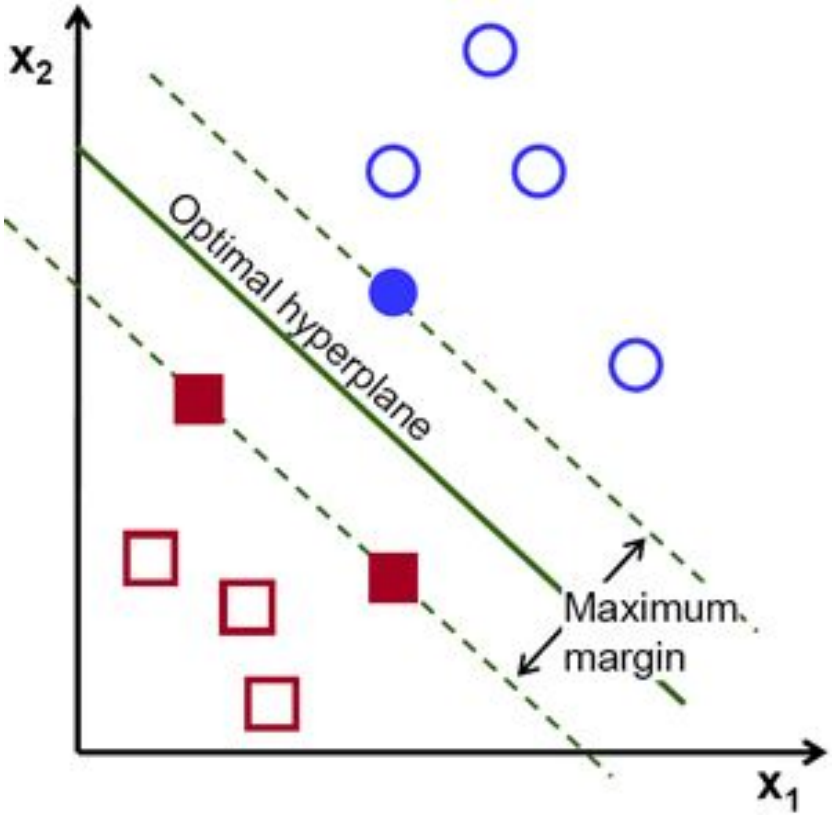
Classify (+) and (-)



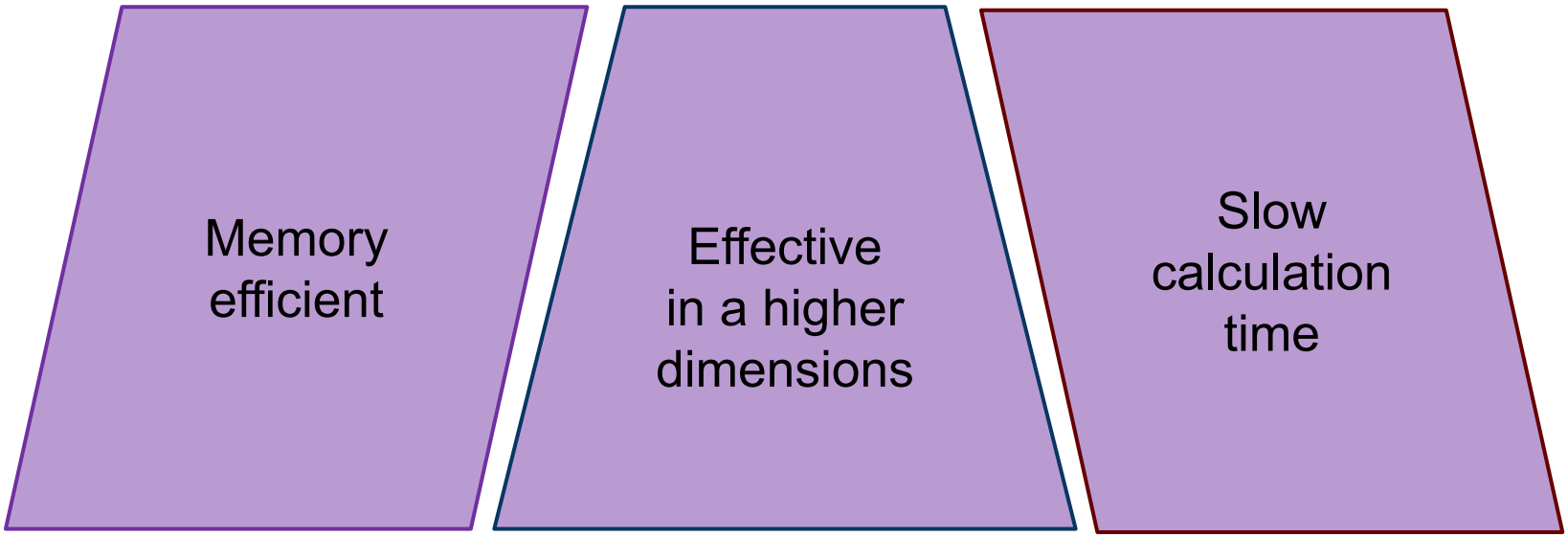
Which Hyperplane?



Optimal Hyperplane

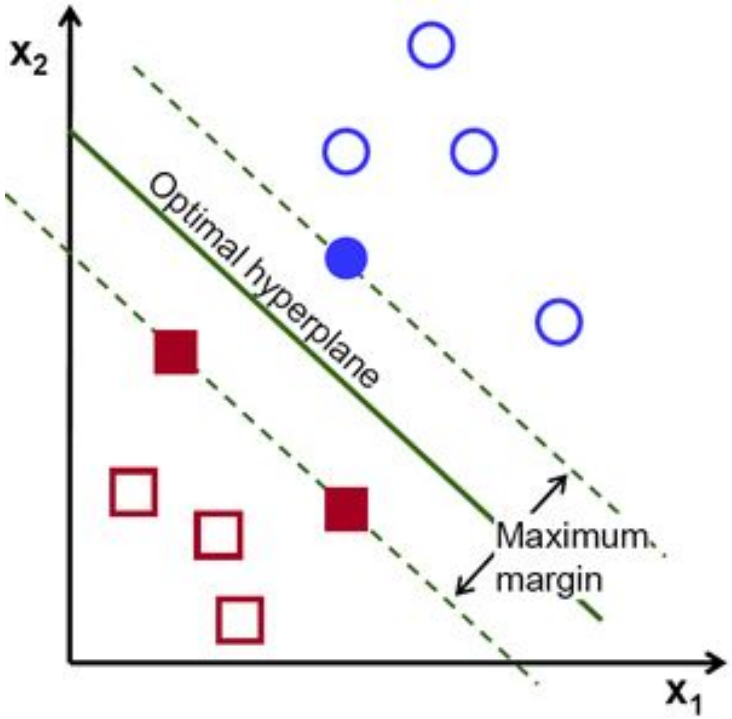


Support Vector Machine

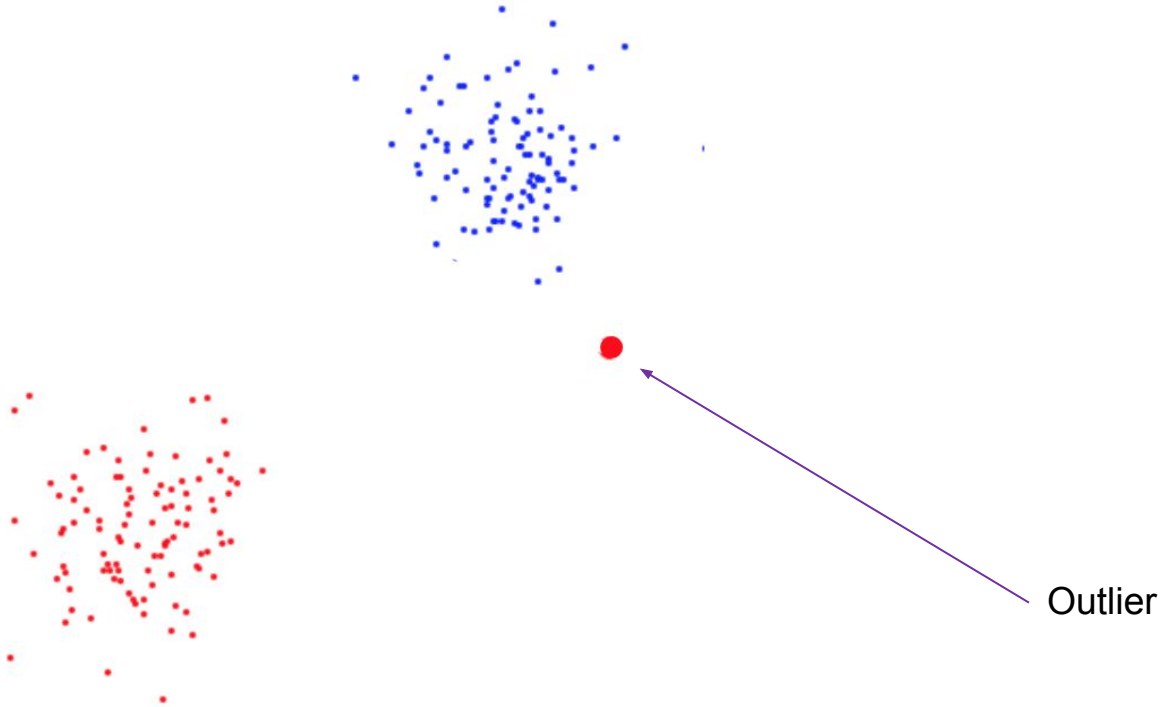


Maximal Margin Classifier

- We want to find a **separating hyperplane**
- Once we find candidates for the hyperplane, we try to maximize the **margin**, the normal distance from borderline points
    - Only **Support Vectors** matter

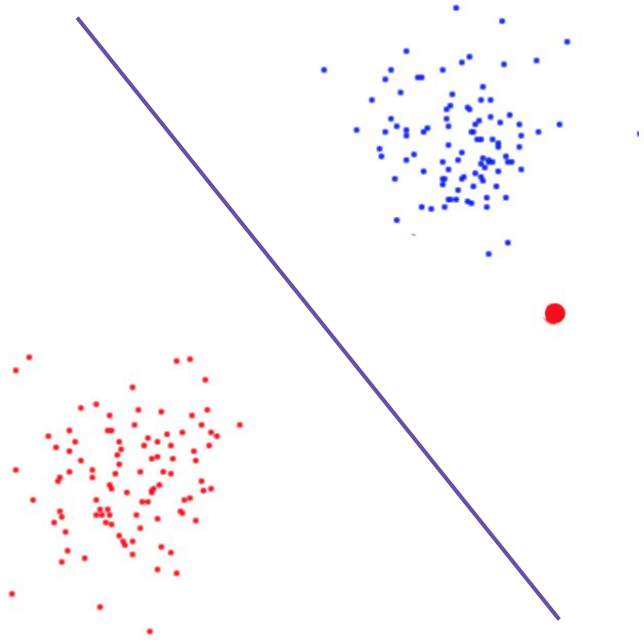


What if…

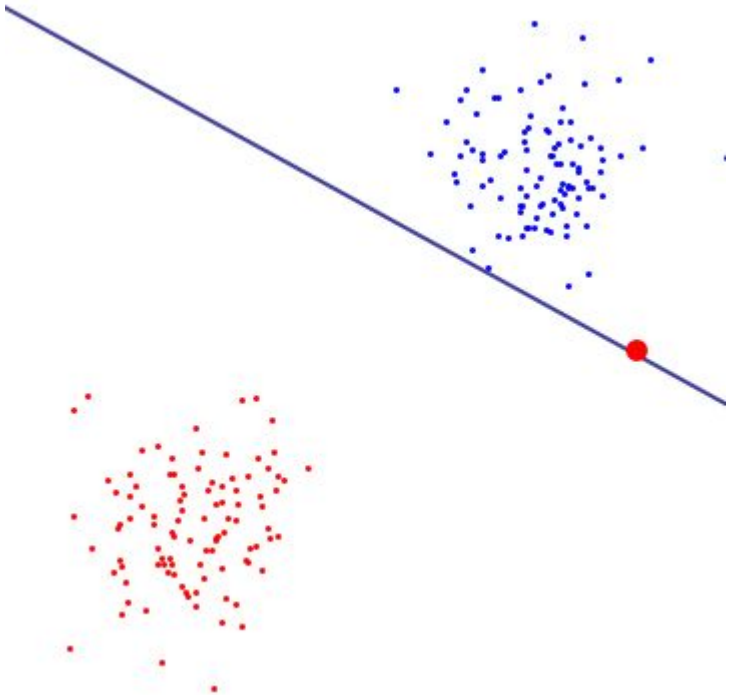


Which Decision Boundary is better?

Boundary 1



Boundary 2



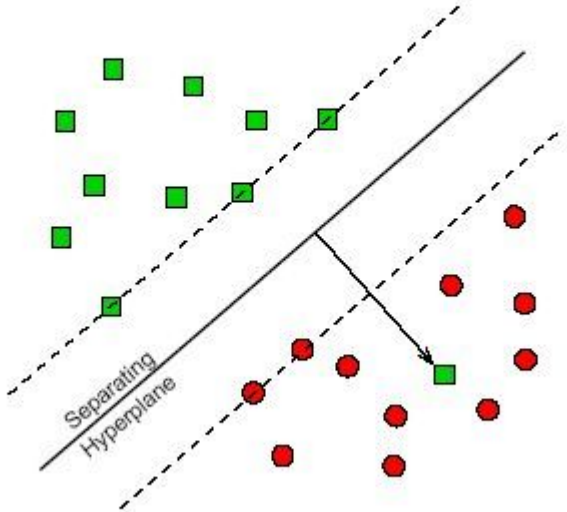
Margins

Use cost function to penalize misclassified points

Choice of cost function makes margin “hard” vs. “soft”

Non-separable training sets

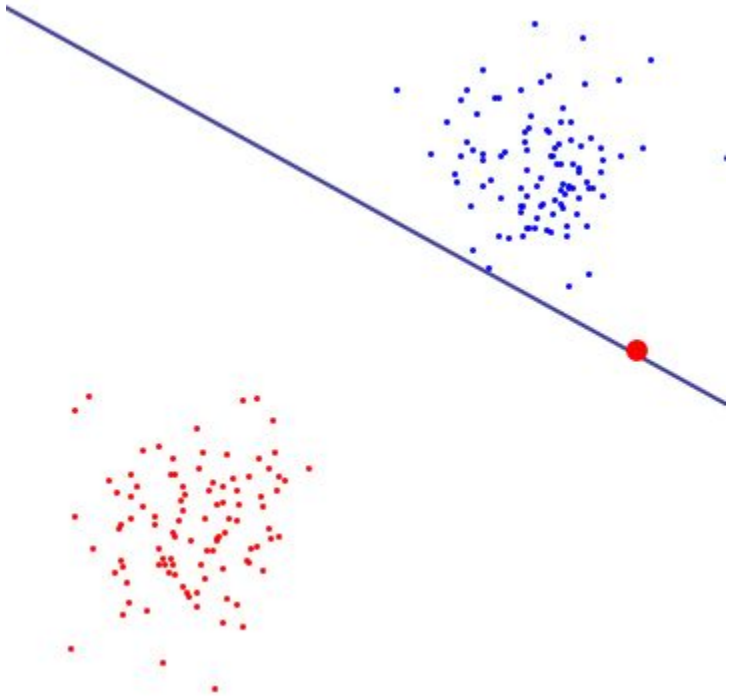
Use linear separation, but admit training errors.



Penalty of error: distance to hyperplane multiplied by *error cost* C.

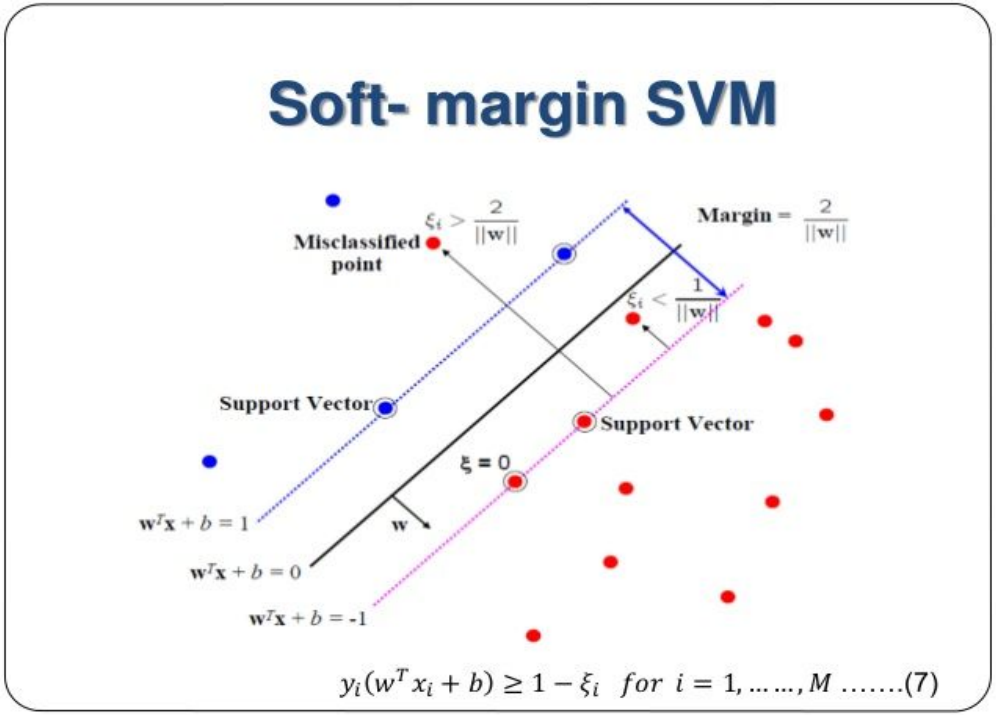
Hard Margins

- High penalty value
- The hyperplane can be dictated by a single outlier

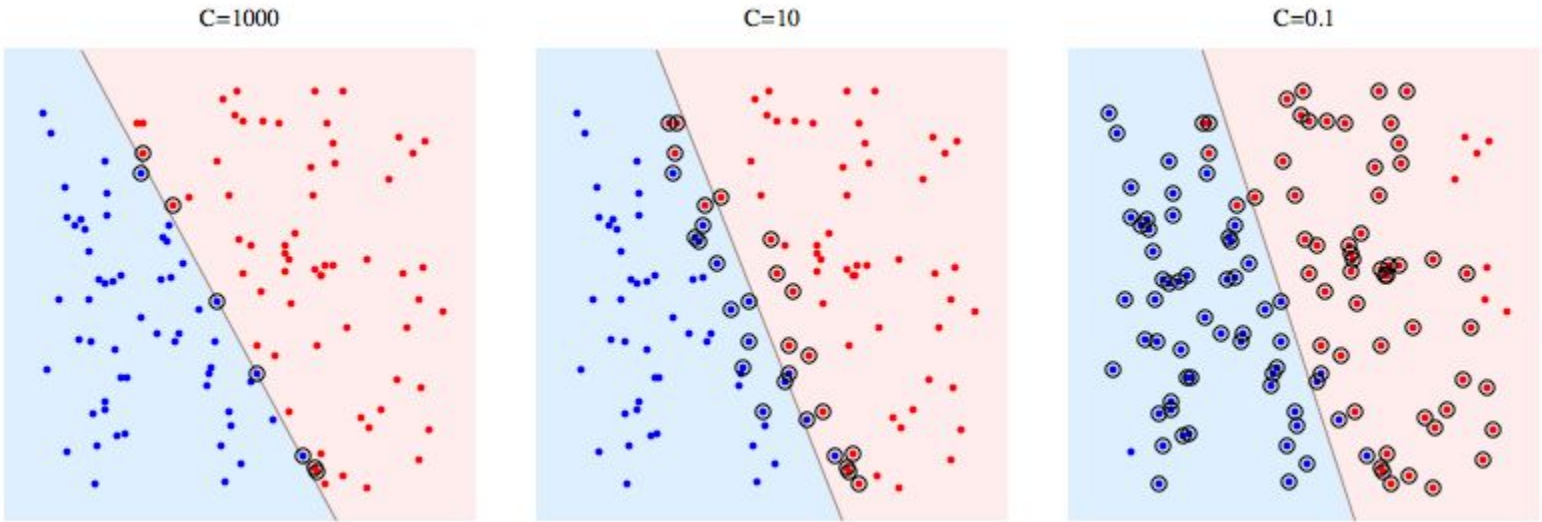


Soft Margins

- Used in non-linearly separable datasets
- Allow for misclassification
- Can account for “dirty” boundaries

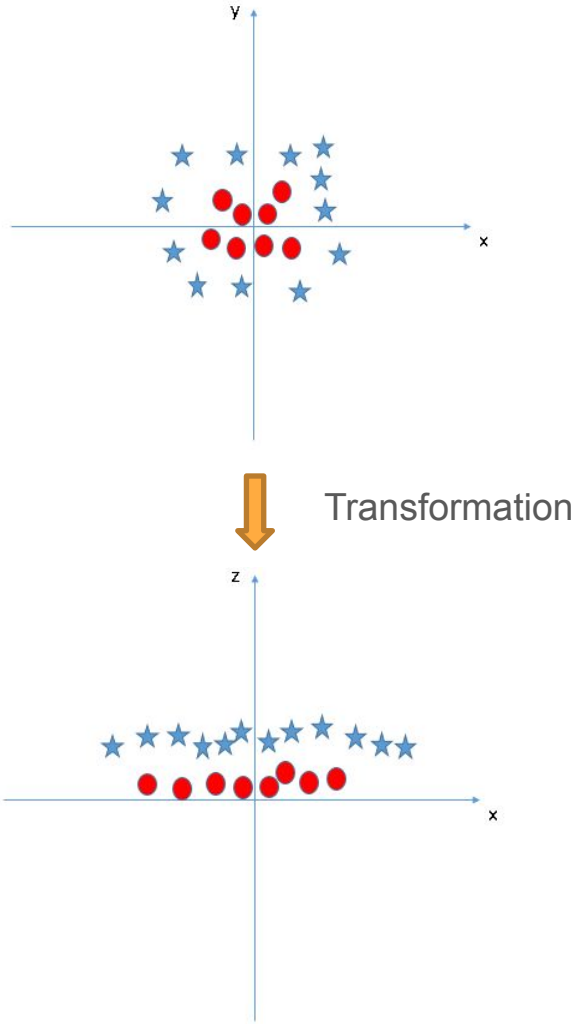


Misclassification Penalty C

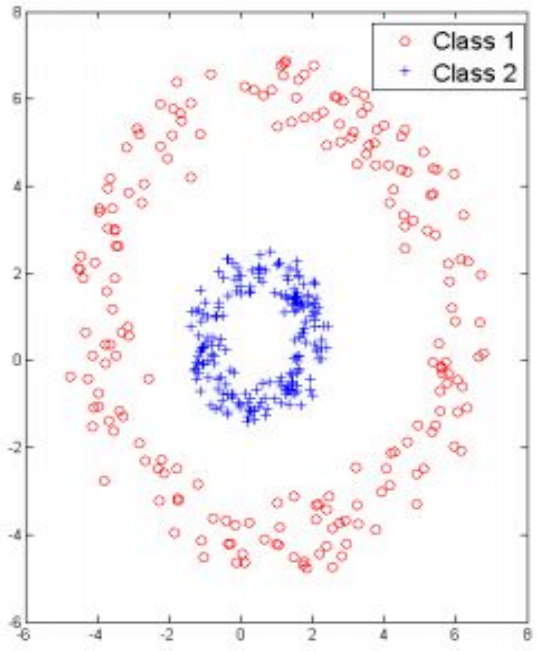


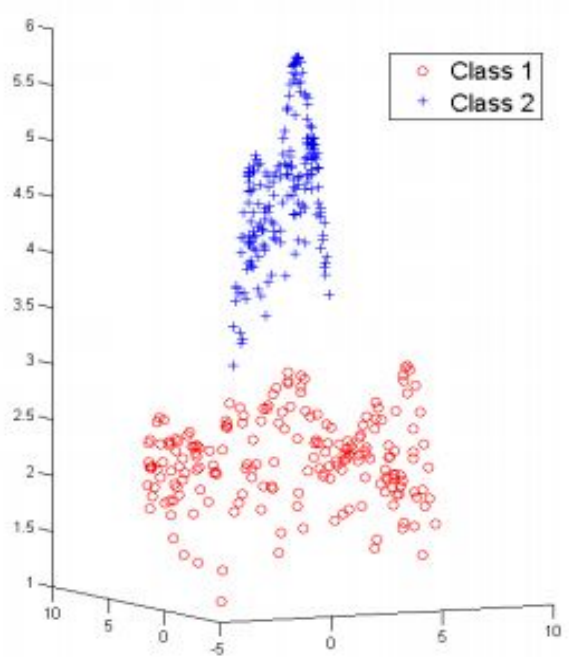
Kernels

- You cannot linearly divide the 2 classes on the *xy* plane at right
- Introduce new feature, $z = x^2 + y^2$ (**radial kernel**)
- Map 2 dimensional data onto 3 dimensional data. Now a hyperplane is easy to find.



Kernels





SVM has MANY Hyperparameters

SVM

C

The “penalty cost” for misclassifications (soft margins)

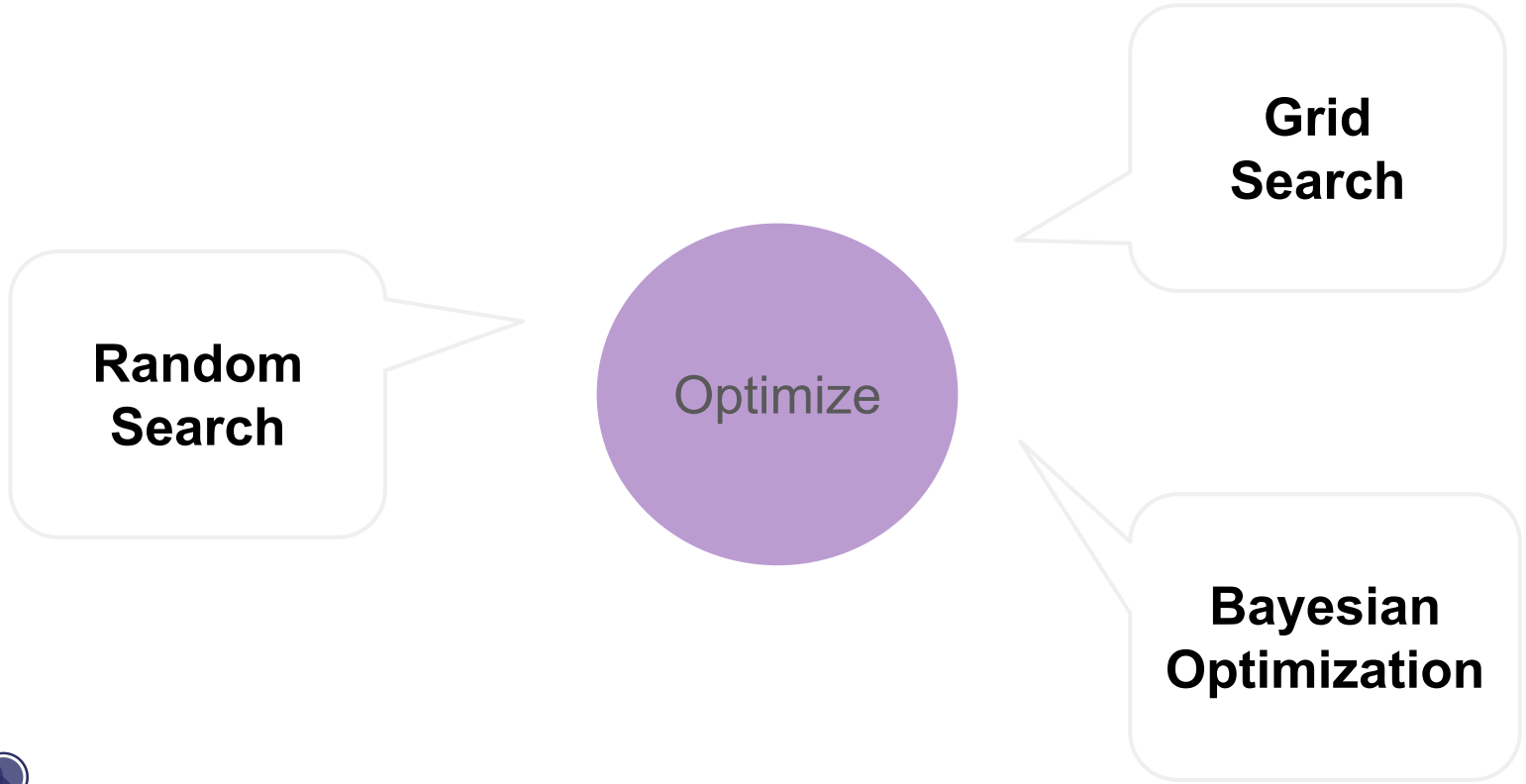
Gamma

How far the influence of a single training example reaches

Kernels

Method of transforming our data set

Finding the Best Hyper Parameters



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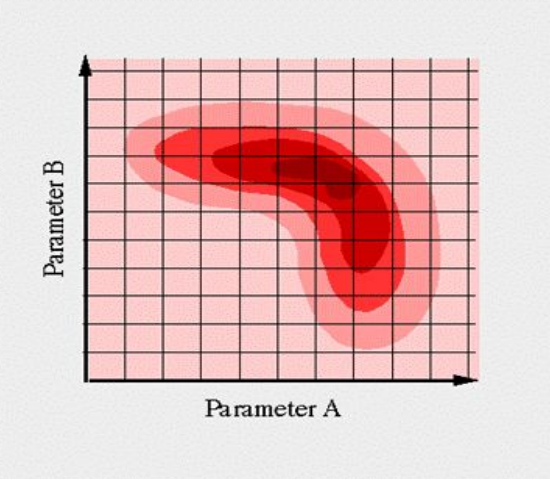
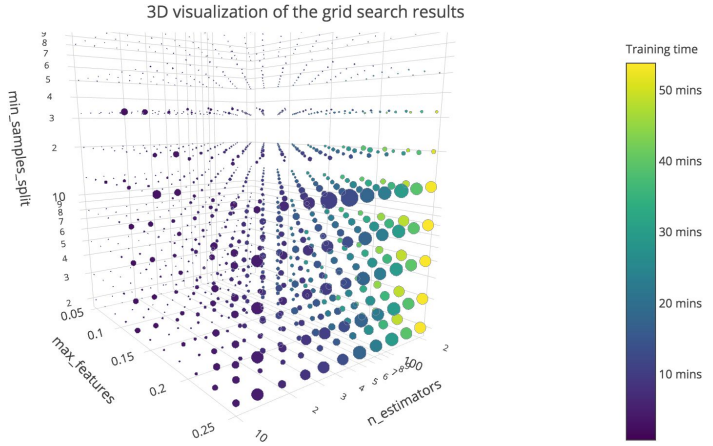
Curse of Dimensionality

Our search space for the optimal hyper-parameters increases **exponentially** as the number of hyper parameters we are considering increases



Add dimension



Overview

Perceptron	SVM
• A very simple model • Will perform poorly if data is not linearly separable	• More complex model because we have to choose the “penalty cost” associated with misclassifications • Can transform feature space by choosing a Kernel

Demo

Cross Validation

K-fold Cross Validation

Often used in practice with *k*=5 or *k*=10.

Create equally sized *k* partitions, or **folds**, of training data

For each fold:

- Treat the *k-1* other folds as training data.
- Test on the chosen fold.

The average of these errors is the validation error

K-fold Cross Validation

Dataset

Suppose K = 5,

5-Fold CV

K-fold Cross Validation

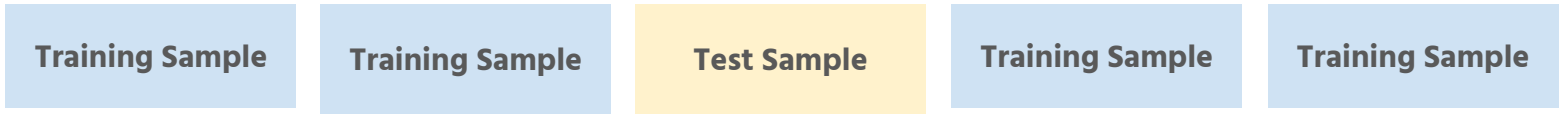
K-fold Cross Validation

Calculate MSE = mse1

K-fold Cross Validation

Calculate MSE = mse2

K-fold Cross Validation



Calculate MSE = mse3

K-fold Cross Validation

And so on

K-fold Cross Validation

MSE = Avg(mse1...5)

K-fold Cross Validation

Matters less how we divide up

Selection bias not present

Leave-1-Out Cross Validation

For each sample:

- Treat all other data as training data.
- Test on that one sample

The average of these errors is the validation error

Pro: Better on small datasets

Pro: More realistic (trained on most of the data)

Con: Takes longer to run

Coming Up

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